In today's context the realities of the Yolŋu world include the immense (and in many respects overwhelming) forces of the Western world, one of these being "education". Within the Western education system educators are struggling to impart to Yolŋu an exotic system of world ordering in the form of mathematics and science curricula. It is a system few Yolŋu acquire, and there is a multitude of reasons as to why this might be so. It is not my purpose here to analyse the failure (nor to argue about who it is that is failing). Rather, I have selected Aboriginal teacher education as a situational context in which to present and illustrate some of the educational implications which emerge from the body of this paper. The recent rethinking of the mathematics curriculum for Aboriginal teacher education students at Batchelor College makes this a dynamic context.

Aboriginal Teacher Education

During the past two years the Teacher Education Mathematics Curriculum at Batchelor College has been radically revised. Previously the curriculum was squarely based on the teaching of Western mathematics, quite divorced from the social context of Aboriginal communities. Stanton (1990a) describes the orientation of the new curriculum (to be implemented in 1991) which promotes the exploration of Western mathematics as a socially negotiated and culturally contextualised meaning system. It seeks to identify ...

ways which de-mystify and make (Western) Mathematics accessible to Aboriginal teacher and child alike; ways which allow the the Aboriginal community to co-opt (Western) Mathematics, its symbolic technology and machines for their own purposes.

A primary focus on the new curriculum will thus include community based research where students will ...

look for and celebrate the mathematics which is to be found ... 'frozen' in Aboriginal tradition. ... Whether or not they may come to identify bridges between their own cultural ways of knowing and that of M.T. (Mathematico-Technological) mathematics will be subject to their own professional judgement.

The curriculum specifically draws a distinction between two meanings of the word mathematics. One refers to the mathematics of the Mathematico-Technological or Western culture, and the other to mathematics from a pan-cultural perspective.
Within this paper three operational definitions of mathematics have been described (p 4-5), with two of them conforming to the distinction made by Stanton. It is my contention that the third perspective on mathematics (i.e Yolŋu mathematics as a system), has been neglected through being confused with the pan-cultural perspective.

For the purpose of maintaining clarity in the discussion which follows, the three kinds of mathematics I refer to, are summarised below:

Definition 1. **Western mathematics**
Mathematics is a Western schema for ordering, patterning, relating and operating on, quantities, based in a European system of logic, where this schema is applied in ordering social activity and in seeking to understand, explain and define the structure and behaviour of whatever exists.

Definition 2. **Pan-cultural mathematics**
Mathematics is the manifestation of a universal human cognitive characteristic reflected in the behaviour of people as they locate in time, space and society. (The categorisation of any given behaviour or linguistic item as mathematical in quality, is based on the presence of a clear conceptual link with Western mathematical notions.)

Definition 3. **Mathematics as a cultural schema.**
Mathematics is a culture's system for encoding, interpreting and organising the patterns and relationships emerging from the human experience of physical, spiritual and social phenomena.

It follows from the discussion of the Yolŋu system of *gurrutu* that it can be defined as mathematics under the terms of Definition 3.

It follows from the investigation of Yolŋu language and social activity in terms of conceptual links with Western mathematical notions of time, measurement, space, number and money, that Yolŋu culture displays a recognisable and significant mathematical aspect according to the terms of Definition 2.

The clearest evidence of Yolŋu abstract cognitive sophistication of a recognisably Western mathematical nature can be identified in Yolŋu kinship structure and in Yolŋu art. However both Yolŋu kinship and art are embedded in and corporate with a cultural schema based in a system of meaning which is alien to the European framework of rational and empirical reasoning. (Thus the foundation of this Yolŋu system of meaning is described in English in terms of "dreamtime myths and legends".) Therefore they cannot be held to indicate a mathematical culture under the terms of Definition 1.
Care must be taken to observe that the statements above which assert that Yolŋu culture has mathematics under the terms of both Definitions 2 and 3, does not imply that mathematics (Definition 2) and mathematics (Definition 3), are equivalent or synonymous.

This distinction, though often overlooked, is by no means trivial and is drawn for its specific educational implications. The investigation by Yolŋu students of traditional ways of locating in time and space or the use of recursion in kinship schema will certainly result in the opportunity to identify a multitude of conceptual connections with Western mathematical notions, as the preceding pages demonstrate. (However the extent to which students can identify these links is of course dependent on their understanding of these Western mathematical notions at the time.) Consider for a moment the example of the Yolŋu painting depicted in figure (11). In connecting it with Western mathematical notions by identifying the geometrical features, the student is engaged in revealing mathematics of the pan-cultural type (Definition 2). However it needs to be stressed that research aimed at the identification of links between the Western and Yolŋu mathematics through this or any other significant painting, does not necessarily imply an investigation of the Yolŋu mathematical system (Definition 3) in which the design is embedded. This Yolŋu mathematics is researched through exploring the systematic links between realms such as creation history, clan identities, totemism and ceremonial song cycles.

I offer a hypothetical and analagous scenario by way of illustration. Imagine that it suddenly became recognised as vitally important for Yolŋu gurrutu to be taught in Balanda schools. Balanda student teachers would be then faced with having to learn to understand Yolŋu gurrutu so that they could teach it. In seeking to understand it they might research into their own Balanda culture looking for a Balanda conceptual system to relate gurrutu to. With support and guidance from elders who are mathematicians, from Yolŋu lecturers and from other Balanda who have learned to grasp a few of its conceptual features, these students would come to identify their own mathematical system as a bridge to understanding gurrutu. Over several years students would be busy learning the special language of gurrutu from Yolŋu lecturers as they struggle to fit the features into mathematical patterns. In doing so they might develop schematic diagrams (such as in figures 1 to 10) presenting complex and elegant patterns which link gurrutu to their existing mathematical understandings.

The question is, how much of an exploration does this process require of Balanda (Western) mathematics? Though some Western mathematical knowledge is important for Balanda in understanding gurrutu, it hardly requires significant research into the philosophy and depths of this Western system. In this hypothetical scenario there is no need to delve into classical or propositional logic, calculus, irrational number theory, or
algebra. Such an exploration has its own importance in the context of a Western education, but it may prove a distraction if the primary purpose is to master the principles of the gurrutu system.

Conversely Yolŋu who seek to understand Western mathematics through relating it to gurrutu, do not need to uncover gurrutu as a whole Yolŋu system of philosophy and meaning. The more instructive parallels can be drawn between the two systems at the more superficial level of gurrutu structure. (As in the analogy above, it is at shallow levels that the teaching potential of one system to the other is found.) Note that this is not intended to belittle the exercise. (On the contrary I can attest to the fact that investigation of the mathematics of gurrutu is demanding, painstaking and intellectually challenging work.) It is simply to point out that the gurrutu system extends in breadth and depth into realms which are beyond the reach of Western mathematics (realms which would be of more help to Yolŋu in relating their world of gurrutu to Balanda ideas about mysticism, spirituality and religion, rather than mathematics).

Within Batchelor College’s new Mathematics Curriculum document for Teacher Education, Stanton presents as a major theme:

A focus on mathematics and its social contexts, which includes the relationships of mathematics to Western science and technology, as well as the notions of a variety of other valid mathematical forms that are to be found in culturally specific situations, especially Aboriginal. (Stanton, 1990b)

This clearly indicates Western mathematics (in its social context) as one focus. It also specifies a consideration of Aboriginal examples of mathematical forms of the pan-cultural type (Definition 2). However although this theme could perhaps be interpreted as also encompassing a study of Aboriginal mathematics in its cultural context (Definition 3), subsequent elaboration in the section headed Research into Traditional Activities, does not support this interpretation:

They (students) would look at, for example, how their people traditionally locate in space, time and society and the use of recursion in kinship schema.

(In support of this kind of student activity Stanton refers to research into the mathematical ideas inherent in the design work of Mozambican artisans proving that they, “like every other people, did mathematics”. He proceeds:)

By providing opportunities for Aboriginal teachers to do similar investigations within their own culture, the stage is set for

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Aboriginal Australia to identify with, co-opt and use (Western) Mathematics.

It thus becomes clear that the notion of Aboriginal mathematics employed in this curriculum, is appropriately categorised under the heading of pan-cultural mathematics. It does not purposefully promote a study by students of their Aboriginal mathematics (Definition 3) in its own context and in its own right. Rather, the purpose is maintained as being the provision of opportunities for students to identify/create their own bridges towards a Western mathematical understanding (this being prerequisitory for the co-opting of this mathematics for their own societal purposes).

The potential for the curriculum to promote the development of significant, meaningful and useful understandings of Western mathematics through the wealth of material which lies embedded (but accessible to Aboriginal students) within Aboriginal cultures is not in doubt. In establishing the parameters (with input from the lecturer) which enable students to identify what is appropriately designated as mathematical, and what is not, and from applying these criteria to cultural contexts over which they have control, they have a genuine opportunity to develop these understandings about the nature of Western mathematics.

This paper has amply demonstrated the capacity for Yolngu language and social structures to sustain a detailed and revealing mathematical examination. And indeed this kind of approach, where the Yolngu world is examined by Yolngu students from Western mathematical perspectives, is already proving useful. I was involved in Teacher Education regional mathematics workshop at Galiwin'ku (March 1990) where kinship structures were examined in this way. One student (Biritjalawuy Gurrwiwi) evaluated the process in the following (translated) words:

During the workshop at Galiwin'ku we were looking at Mathematics in a Yolngu way. We were bringing out into the open mathematics that normally lies hidden in our world. ....

Mathematics can be found all over the place. For example when we say the word walu (sun) we can mean two things. One meaning for the word walu is that hot ball of fire which shines in the sky. But there are other meanings as well, such as wristwatch, and it also means day. These relate to the concept of time. This is also mathematics.

Then we turned to look at our kinship system, at our relations (gurrutu) and subsections (mälk). Don't think that we were trying to learn our kinship system because we learnt that a long time ago and still remember. Instead we were looking at the mathematics
which lies inside the kinship system. We wrote down all our kinship terms onto paper and then drew lines between them to see the patterns.

First we looked for ourselves at what we had done from the mathematics side. Then Michael put together these patterns that we saw into the one diagram (similar to Yolŋu Kinship Patterns: figure 6) as you can see. So what you can see on the paper is like how Balanda see the mathematics there.

When you look at the diagram don’t expect to be able to recognise everything straight away. Take your time and look at it carefully, and then you will understand it after a while.

We don’t want to take that diagram and teach that to our children at our school here. Instead this diagram shows us how Yolŋu can identify that “mathematics” which is a Balanda idea. It also shows us how Balanda look at our kinship system.

The last paragraph raises a complex issue of vital concern to educators. Whilst seeing value in learning about Western mathematics through Yolŋu kinship, the Yolŋu writer is clearly alarmed at the thought of children learning gurrutu through Western mathematical processes. The social upbringing of Yolŋu children is centred on instilling within them their social identity and their way of relating to all that is around (people, wilderness and land). It is an undertaking that is not appropriately left to the drawing of diagrams in the classroom. It is not seen as objective knowledge that one learns about. The knowledge becomes the child in the formation of the child’s identity. In other words Yolŋu mathematics should be taught in Yolŋu contexts and in Yolŋu ways. This concern brings us back to the issue of determining the place of Yolŋu mathematical education (Definition 3) within the Yolŋu school, and by implication within Batchelor College. As Yolŋu schools move closer to the goal of Aboriginalisation of the teaching staff, parents look to their Yolŋu teachers to share responsibility for the Yolŋu enculturation of children. In accepting this responsibility Yolŋu teachers need to be sure of this knowledge themselves and they also need to have developed strategies adapted for the cultural education of children in the school situation.

The need for children to learn Yolŋu mathematics (Definition 3) is recognised by Yolŋu elders. The advice of elders working with one group of Yolŋu researchers at Yirrkala is reported as follows (in Raymattja et al, 1990):

Children in our schools must progress along two ‘raki’ (lines of conceptual development). This is particularly important in the abstract conceptual areas, such as those covered by mathematics
in the conventional *Balande* School curriculum. Children in our schools must learn to use the dual systems of abstract understanding: those of the *Yolŋu* world and those of the *Balande* world.

Providing Yolŋu children with a schooling which can achieve this aim of the development of two divergent systems of abstract understanding is a formidable challenge. Perhaps the greatest difficulty lies in preventing the interference of one with the other during the intellectual, linguistic and cultural development of the child. Thus most important decision for a school community is in negotiating the meaning of "both-ways" education. In one community it might mean maintaining a clear separation of the two systems of abstract understanding as areas within the school curriculum (where the bridge building between the two systems is left to material that falls within Definition 2). In another it might mean a continual process within the school curriculum of exploring and making explicit the tensions between the systems, and searching for points where they can meet each other meaningfully and productively.

Either way there are significant opportunities afforded by the order and patterns within Yolŋu social and cultural life, for introducing Yolŋu children into the Western system of abstract understanding. Thus for example, in examining the Western mathematical notion of division, there could be a clear distinction negotiated between the sharing of turtle eggs and the sharing of damper (where damper is not broken into equal pieces and is not normally given out on the basis that each gets a share of equal mass/size).

However in using Yolŋu content teachers must be clear about what it is that they are (and are not) teaching. Holding up recursion patterns within the Yolŋu kinship system or geometry within Yolŋu paintings, provides Yolŋu examples of pan-cultural mathematics. (In the case of Yolŋu art this is achieved by setting aside the Yolŋu meaning of their graphic symbolic system.) Of course the teacher is not prevented from digressing into realms of Yolŋu mathematical schemata, but one would expect that work on kinship from Yolŋu perspectives, using Yolŋu processes, would *precede* an analysis of patterns; and that work on the Yolŋu symbolism within Yolŋu art would *precede* its geometrical analysis.

There are other potential difficulties for Yolŋu teachers in taking children into the *gurrulstu* system from the Yolŋu mathematical perspective (as opposed to the pan-cultural mathematical perspective). One is that it may not be recognised by others for the legitimate study of mathematics that it is. (Non-Aboriginal educators may see it as Social and Cultural Education, which it also is.) Another is that the Yolŋu teacher may not be fully confident of his or her own understanding of
gurruṯu as an abstract system. It is the Yolŋu teachers who are constantly faced with resolving the tension between the knowledge systems in the developing minds of the children. To be successful the teachers must have already worked through these tensions within themselves. They must be confident in the definition which they have negotiated for both-ways education within their communities. They must also work through the meanings of mathematics in negotiating a system of mathematics education which will support their chosen form of both-ways education.

Thus the role of Batchelor College as a teacher training institution is critical. Teacher Educators must address as an issue, the place of Yolŋu mathematics (Definition 3) as a discrete focus within the Mathematics Curriculum. Younger adult students are the first to acknowledge that they are not masters of gurruṯu theory. Even grey haired adults will explain that there are white haired elders whose knowledge runs deeper. There are changes in the fabric of contemporary Yolŋu societies caused by the effects of non-Aboriginal forces which make the pathway to this knowledge more difficult to follow. The College’s responsibility to cater for individual students who express a need to deepen their own knowledge in an area as important (and sometimes as private) as Yolŋu mathematics, needs to be fully acknowledged.

Student research into Yolŋu mathematics (Definition 3) as part of students' own personal and professional development, does take place on occasion in the context of community based Aboriginal language workshops, where Batchelor College's Yolŋu students may negotiate individual projects involving learning from elders, and which constitute investigations into the order which underlies aspects of their culture.

As a group, or across group activity within the College's Mathematics Curriculum, it as an area of adult learning which would require clear understandings on the part of all concerned. Findings can only be presented in the most superficial terms using English (unless perhaps participants share an extensive command of specialist anthropological vocabulary). There are sacred aspects which prevent discussion at deeper levels in mixed-sex, mixed-age groups of students, and discussion at any significant depth excludes all but the most unusual (non Yolŋu) tutor or lecturer. Non-Aboriginal lecturers would need to accept the probability that this emergent Yolŋu mathematics would not be recognisable to them as mathematics, because of the dominance of their Western mathematical perspectives. Students and lecturers would be faced with the need to re-assess priorities within the Mathematics Curriculum in determining how much time and space should be given to Yolŋu mathematics. Whilst the study of pan-cultural Yolŋu mathematics is demonstrably proximate to Western mathematics and thus valuable in leading to understandings of Western mathematics, the study of the distinct Yolŋu mathematical schema as a discipline in itself, is not so
easily accepted as being related to conventional goals of mathematics education.

The promotion of student research into the Yolgnu mathematical schema of *gurrutu* would require a readjustment of the College's set patterns for individual student research. This might involve the provision of funds to pay for the time and expertise of elders (whose minds are the libraries of Yolgnu society); provision of time within the curriculum; and a rethinking of assessment procedures. As individuals and as members of Yolgnu school communities, Yolgnu students, in seeking strength through knowledge, require from Batchelor College acceptance of and support for the various paths which their different school communities are forming in their attempts to both maintain their cultural strength as Yolgnu, and to take from the Western domain of knowledge that which they need. It is the Yolgnu teachers who as mediators, must have the clearest perspective of each side, which is gained from acquiring confidence in the two systems of abstract understanding, held in two mathematics (Balanda and Yolgnu), and from exploring (or creating) commonalities through a third (pan-cultural mathematics).